CLASS XII PHYSICS · CHAPTER 4

Moving Charges and Magnetism

Revision Deck — Lorentz force, Biot-Savart law, Ampere's law, solenoids, torque on a current loop, and the moving-coil galvanometer.

Formulas

Worked Examples

Key Concepts

Lorentz Force & Force on a Conductor

Total force on a moving charge

$$F = q\,[\,E + v \times B\,]$$

Magnetic part: $F = qvB \sin\vartheta$, ϑ = angle between v and B

Force on a current-carrying rod

$$F = I\,(\,L \times B\,),\quad F = B \times I \times L \sin\theta$$

L points along the direction of current I

- Direction: perpendicular to both v and B — given by the right-hand (screw) rule.
- Force on a negative charge is opposite to that on a positive charge.
- $F = 0$ when $v \parallel B$ or $v \perp B$ is false — i.e. $\sin\theta = 0$ (parallel/antiparallel), or when $v = 0$.
- Magnetic force does no work — it changes direction, never speed.

Unit of B

$$1\ T = 1\ N\ s\ C^{-1}\ m^{-1}$$

1 gauss (G) = 10^{-4} T | Earth's field ≈ 3.6×10^{-5} T
B is the SI unit when a 1 C charge moving at 1 m/s ⊥ to B feels a 1 N force.

Circular & Helical Motion of a Charge

$\theta = 0°$ or $180°$

v ∥ B (or antiparallel)

$F = 0$

Charge moves undeflected.

$\theta = 90°$

v ⊥ B

$r = mv/qB$

Uniform circular motion; qvB acts as centripetal force.

$0° < \theta < 90°$

v has components ∥ and ⊥ to B

Helical path

∥ component unaffected; ⊥ component circles.

Angular frequency & period

$$\omega = 2\pi\nu = qB/m\,,\ \ T = 2\pi m/qB$$

Independent of speed/energy — basis of the cyclotron.

Radius of circular path

$$r = mv\ /\ qB$$

Larger momentum ⇒ larger radius.

Pitch of helix

$$p = v_\| T = 2\pi m\, v_\|\ /\ qB$$

Distance moved along B in one revolution.

Exam tip: Cyclotron frequency $\nu = qB/2\pi m$ does NOT depend on speed r energy — this independence is exactly what makes the cyclotron work.

The Biot–Savart Law

Vector form

$$dB = (\mu0/4\pi) \cdot I\, dl \times \hat{r} / r^2$$

$dB = (\mu0/4\pi) \cdot I\, dl\, \sin\vartheta / r^2$ (magnitude)

Permeability of free space

$$\mu0 = 4\pi \times 10^{-7}\ \mathrm{T\ m/A}$$

$\mu0/4\pi = 10^{-7}$ T m/A (exact constant)

Biot–Savart Law vs Coulomb's Law

Aspect	Coulomb's Law (E)	Biot–Savart Law (B)
Source	Scalar (charge q)	Vector (current element I dl)
Range	Long range, $\propto 1/r^2$	Long range, $\propto 1/r^2$
Direction	Along displacement vector r	$\perp$ to plane of dl and r
Angle dependence	None	$\propto \sin\theta$

Relation to light: $\mu0\ \varepsilon0 = 1/c^2$ → fixing $\mu0 = 4\pi\times10^{-7}$ T m/A fixes $\varepsilon0$.

Magnetic Field of a Circular Current Loop

On the axis, distance x from centre

$$B = \mu 0\ I\ R^2 / [\ 2(x^2 + R^2)\text{^}(3/2)\ ]$$

R = radius of loop, I = current

At the centre of the loop (x = 0)

$$B = \mu 0\ I / 2R$$

Maximum field value on the axis

- Direction: right-hand thumb rule — curl fingers along current direction; thumb gives B.
- Upper face with anticlockwise current behaves like a N-pole; lower face (clockwise) like S-pole.
- For N tightly wound turns, multiply B by N.
- Field lines form closed loops through the coil (like a bar magnet's).

Far-axis limit ($x \gg R$) — magnetic dipole

$$B = (\mu 0/4\pi) \cdot 2m/x^3\ ,\ \ m = IA$$

m = magnetic moment. Compare with electric dipole field $E = (1/4\pi\varepsilon 0)\cdot 2p/x^3$ — replace $m \leftrightarrow p$, $\mu 0 \leftrightarrow 1/\varepsilon 0$, $B \leftrightarrow E$.

Symmetry Shortcut for the Biot–Savart Law

General statement

$$\oint B \cdot dl = \mu 0 \ Ienc$$

Line integral of B around any closed (Amperian) loop equals μ0 × current enclosed.

Simplified form (B tangential & constant on loop)

$$B \ L = \mu 0 \ Ienc$$

Sign convention: right-hand rule on the loop traversal.

Field of a long, straight current-carrying wire

Outside the wire (r > a)

$$B = \mu 0 \ I / 2\pi r$$

$B \propto 1/r$

Inside a uniform wire (r < a)

$$B = (\mu 0 \ I / 2\pi a^2) \ r$$

$B \propto r$ (a = wire radius)

Direction

Right-hand rule

Thumb along current; curled fingers give B — concentric circular field lines.

Note: Ampere's law is not independent of Biot–Savart law — it can be derived from it. Their relation mirrors that between Gauss's law and Coulomb's law. It only simplifies calculations when the setup has enough symmetry.

Uniform Field from a Long Coil

Field inside a long solenoid

B = μ0 n I

n = N/l = turns per unit length, uniform & parallel to axis

Amperian loop used

B l = μ0 (n l) I

Rectangular loop abcd; only side ab (inside, ∥ to axis) contributes.

- Field between neighbouring turns cancels out; field outside the solenoid ≈ 0.
- Long solenoid ⇒ length ≫ radius, so the idealisation B_outside = 0 holds well.
- Direction of B given by the right-hand rule.
- A soft-iron core inside greatly increases the field strength (used in galvanometers).

Worked check

n = 500/0.5 = 1000 turns/m
B = μ0 n I = 4π×10^{-7} × 1000 × 5

= 6.28 × 10^{-3} T (l = 0.5 m, r = 1 cm, N = 500, I = 5 A)

The Ampere, Defined

Force per unit length between two wires

$$f = \mu_0 I_a I_b / 2\pi d$$

d = separation between wires a and b

Definition of 1 ampere

If $I_a = I_b = 1$ A, $d = 1$ m

then $f = 2 \times 10^{-7}$ N/m — this fixes the SI unit ampere.

Parallel currents
ATTRACT

Antiparallel currents
REPEL

- Opposite of electrostatics, where like charges repel. Biot–Savart law + Lorentz force ⇒ results consistent with Newton's third law.
- 1 coulomb = charge flowing in 1 s through a conductor carrying a steady current of 1 A.

Magnetic Dipole Moment

Torque on a planar current loop

$$\tau = IAB \sin\theta = m \times B$$

ϑ = angle between B and the normal to the loop (area vector A)

Magnetic moment

$$m = I A \quad (N \text{ turns: } m = N I A)$$

Direction by right-hand thumb rule; unit A m^2

- Net force on the loop is always zero — only a torque acts.
- $\tau = 0$ when m ∥ B (stable equilibrium) or m antiparallel B (unstable equilibrium).
- A loop of irregular shape carrying current, if flexible, becomes circular — max. flux for given perimeter.

Circular loop as a magnetic dipole ($x \gg R$)

$$B = (\mu 0/4\pi) \cdot (2m/x^3) \quad [\text{axial}]$$

Analogy: m⟷p, B⟷E, μ0⟷1/ε0 (electric dipole formulas carry over)

MCG link: This torque (τ = NIAB) is exactly what a moving-coil galvanometer measures — see next slide.

Measuring Current via Torque Balance

Equilibrium (torque balance)

$$k\,\varphi = N\,I\,A\,B$$

Magnetic torque NIAB balanced by spring's restoring torque $k\varphi$

Deflection

$$\varphi = (NAB/k)\,I$$

$\varphi \propto I$ — the basis for a linear scale

Current sensitivity

$$\varphi/I = NAB/k$$

Deflection per unit current

Voltage sensitivity

$$\varphi/V = (NAB/k) \cdot (1/R)$$

R = coil resistance

If N → 2N:

Current sensitivity DOUBLES ($\phi/I \rightarrow 2\cdot\phi/I$)
Voltage sensitivity UNCHANGED (R also doubles, since $R \propto$ wire length)

- Field is radial (soft-iron core) so $\sin\theta = 1$ always — scale is linear.
- MCG cannot directly read current: full-scale deflection at only ~μA, and its resistance would disturb the circuit.

Ammeter & Voltmeter Conversion

AMMETER

Small shunt resistance S in PARALLEL with coil G

$$S = I_g G / (I - I_g)$$

Effective resistance ≈ S, kept very small so the circuit current is barely disturbed.

VOLTMETER

Large resistance R in SERIES with coil G

$$R = V/I_g - G$$

Effective resistance ≈ R (large), so it draws negligible current and barely disturbs the voltage being measured.

Worked example (I_g = 2.5 mA, G = 12 Ω → ammeter of range 0–7.5 A)

$$S = (2.5\times10^{-3} \times 12) / (7.5 - 2.5\times10^{-3}) = 4 \times 10^{-3}\ \Omega \quad \text{(connected in parallel with the coil)}$$

QUICK REFERENCE

All Key Formulas at a Glance

Lorentz force

$F = q(E + v \times B)$

Force on current-carrying rod

$F = I(l \times B) = BIl \sin\theta$

Radius of circular orbit

$r = mv/qB$

Cyclotron frequency

$\nu = qB/2\pi m$

Biot–Savart law

$dB = (\mu 0/4\pi)\ I\ dl \sin\theta\ /\ r^2$

Field at centre of loop

$B = \mu 0 I/2R$

Ampere's circuital law

$\oint B \cdot dl = \mu 0\ Ienc$

Field of straight wire

$B = \mu 0 I/2\pi r$

Field inside solenoid

$B = \mu 0 nI$

Force between wires (per length)

$f = \mu 0 IaIb/2\pi d$

Torque on current loop

Galvanometer deflection

PRACTICE

Worked Example Problems

1

A 200 g, 1.5 m wire carries 2 A and floats in a horizontal field B. Find B.

Solution:

$mg = BIl \rightarrow B = mg/Il = (0.2\times9.8)/(2\times1.5) = 0.65$ T

2

Electron ($v=3\times10^{7}$ m/s) enters $B=6\times10^{-4}$ T $\perp$ to it. Find r, frequency, KE.

Solution:

$r = mv/qB = 28$ cm; $\nu = v/2\pi r \approx 17$ MHz; $E = \frac{1}{2}mv^2 \approx 2.5$ keV

3

Solenoid: l=0.5 m, N=500 turns, I=5 A. Find B inside.

Solution:

$n = N/l = 1000/\text{m} \rightarrow B = \mu0nI = 4\pi\times10^{-7}\times1000\times5 = 6.28\times10^{-3}$ T

4

100-turn coil, r=10 cm, I=3.2 A. Find B at centre and magnetic moment m.

Solution:

$B = \mu0NI/2R = 2\times10^{-3}$ T; $m = NI\pi r^2 = 10$ A m^2

BEFORE YOU WALK IN

Points to Remember

- Magnetic field lines always form closed loops — unlike electric field lines, which start/end on charges.
- Ampere's circuital law is derivable from Biot-Savart law; use it only where symmetry makes B tangential/normal/zero on a chosen loop.
- $\tau = m \times B$ and $F = q(v \times B)$ both involve cross products — direction questions are common; practise the right-hand rule.
- Magnetic force never does work on a charge — only redirects its velocity.
- Distinguish current sensitivity (ϕ/I) from voltage sensitivity (ϕ/V): increasing N doesn't always increase both.